

# Constraints on the DGP Universe Using Observational Hubble parameter

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In this work, we use observations of the Hubble parameter from the differential ages of passively evolving galaxies and the recent detection of the Baryon Acoustic Oscillations (BAO) at  $z_1 = 0.35$  to constrain the Dvali-Gabadadze-Porrati (DGP) universe. For the case with a curvature term, we set a prior  $h = 0.73 \pm 0.03$  and the best-fit values suggest a spatially closed Universe. For a flat Universe, we set  $h$  free and we get consistent results with other recent analyses.

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## I. INTRODUCTION

Observations of Wilkinson Microwave Anisotropy Probe (WMAP) [1], the type Ia Supernova (SN Ia) [2, 3] and Sloan Digital Sky Survey (SDSS) [4, 5] support an accelerated expanding Universe. Many cosmological models have been constructed to explain such a cosmology. Most of them concentrate on the dark energy term with a negative pressure, within the usual gravitation theory.

The observed accelerated expansion of the Universe is perhaps due to some unknown physical processes involving modifications of gravitation theory. Such modifications are usually related to the possible existence of extra dimensions, giving rise to the so-called braneworld cosmology. The braneworld cosmology is an example which excludes the dark energy term by modifying the gravitation theory [6, 7, 8, 9]. One interesting braneworld cosmological model is the one proposed by Dvali et al., which is usually called the Dvali-Gabadadze-Porrati (DGP) braneworld [10, 11, 12]. For scales below a crossover radius  $r_c$ , the gravitational force experienced by two punctual sources is the usual 4-dimensional  $1/r^2$  force whereas for scales larger than  $r_c$  the gravitational force follows the 5-dimensional  $1/r^3$  behavior.

Although the theoretical consistency and especially its self-accelerating solution are still waiting for confirming [13, 14], the DGP models have been successfully tested from the observations. Deffayet et al. discussed observational constraints from the Cosmic Microwave Background (CMB) and SN Ia [15]. Jain et al. presented a constraint from the viewpoint of gravitational lenses [16]. Alcaniz et al. used the estimated ages of high- $z$  objects to constrain the cosmological parameters [17]. The Chandara measurements of the X-ray gas mass fraction in galaxy clusters were used to do a combinational analysis with other cosmological probes [18]. Pires et al. tested the viability of DGP scenarios from the cosmological time measurements, i.e., recent estimates of the total age of the Universe and observations of the lookback time to galaxy clusters at intermediary and high redshifts [19]. Guo et al. constrained the DGP model from recent supernova observations and BAO [20]. Zhu et al. did the similar work using SN Ia [21]. See [22, 23] for more corresponding comments on the DGP Universe.

In this work, we examine the DGP Universe using the observational  $H(z)$  data (sometimes we call them OHD for simplicity) [24, 25]. The observational  $H(z)$  data are related to the differential ages of the oldest galaxies, the derivative of redshift  $z$  with respect to the cosmic time  $t$  (i.e.,  $dz/dt$ ) [25]. A determination of  $dz/dt$  provides a measurement of the Hubble parameter, which can be used as an effective cosmological probe. In addition, we do the combinational analysis using data of the size of the Baryonic Acoustic Oscillations (BAO) peak detected in the large-scale correlation function of luminous red galaxies from the Sloan Digital Sky Survey (SDSS) [26]. For a Universe with a curvature term, a prior for the dimensionless Hubble constant  $h = 0.73 \pm 0.03$  is taken from the combinational WMAP three-year estimate [1]. And we find that the best-fit values for both two cases suggest a closed Universe. For a flat DGP Universe, we set  $h$  free and get the results consistent with other independent analyses. The values of the current deceleration parameter, the transition redshift at which the Universe switches from deceleration to acceleration and the current value of the effective equation of state are discussed too.

This paper is organized as follows: In Sec.2, we briefly review the DGP Universe. In Sec.3, we introduce the observational  $H(z)$  data and the BAO data. In Sec.4, we present the constraints on the DGP Universe. Discussions and conclusions are given in Sec.5.

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## II. OVERVIEW OF THE DGP UNIVERSE

The DGP theory has an important parameter  $r_c$  which is the crossover radius where the theory changes between a region that is effectively 4-dimensional to what is fully 5-dimensional. It is defined as

$$r_c = \frac{M_{\text{Pl}}}{2M_5^3}, \quad (1)$$

where  $M_{\text{Pl}}$  is the Planck mass and  $M_5$  is the 5-dimensional reduced Planck mass. In the DGP Universe, the modified Friedmann equation due to the presence of an infinite-volume extra dimension reads [15, 28]

$$H^2 = \left[ \sqrt{\frac{\rho}{3M_{\text{Pl}}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right]^2 - \frac{k}{a(t)^2}, \quad (2)$$

where  $H$  is the Hubble parameter,  $\rho$  is the energy density of the cosmic fluid and  $k = 0, \pm 1$  is the spatial curvature parameter.

If we use the definition

$$\Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}, \quad (3)$$

the Hubble parameter can be rewritten as

$$H(z)^2/H_0^2 = \Omega_k(1+z)^2 + [\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m(1+z)^3}]^2, \quad (4)$$

where  $z$  is the redshift,  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$  is the current value of the Hubble parameter,  $\Omega_m$  and  $\Omega_k$  are the matter and curvature density parameters respectively.

And we can get this relation from the above equation by setting  $z = 0$ ,

$$\Omega_k + [\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m}]^2 = 1. \quad (5)$$

The current value of the deceleration parameter  $q = -\ddot{a}/aH^2$  takes the form [20]

$$q_0 = \left( \frac{1}{2}\Omega_m - \Omega_{r_c} \right) \left( \frac{\sqrt{\Omega_{r_c}}}{\sqrt{\Omega_m + \Omega_{r_c}}} + 1 \right) - \sqrt{\Omega_{r_c}^2 + \Omega_m \Omega_{r_c}}. \quad (6)$$

For a flat Universe with  $\Omega_k = 0$ , Eq.(5) reduces to  $\Omega_{r_c} = (1 - \Omega_m)^2/4$ , from which we get  $0 \leq \Omega_{r_c} \leq 0.25$  for  $0 \leq \Omega_m \leq 1$ . The current value of the deceleration parameter can be written as

$$q_0 = -1 + \frac{3}{2} \frac{\Omega_m}{\sqrt{\Omega_m + \Omega_{r_c}}}. \quad (7)$$

If we define  $s = \Omega_{r_c}/\Omega_m$ , the transition redshift  $z_{\text{tr}}$  at which the Universe switches from deceleration to acceleration can be expressed as [23]

$$z_{\text{tr}} = -1 + 2s^{1/3}. \quad (8)$$

Also, we can derive the DGP Universe expressed in Eq.(4) using the time-dependent effective equation of state [23]

$$\omega_{\text{eff}}(z) = -1 + \frac{1}{2} \frac{(1+z)^3}{s + (1+z)^3 + \sqrt{s} \sqrt{s + (1+z)^3}}. \quad (9)$$

It is clear that  $\omega_{\text{eff}} \rightarrow 0.5$  at  $z \rightarrow \infty$ . The current value of  $\omega_{\text{eff}0}$  depends on  $s$ , and is always larger than -1.

## III. THE OBSERVATIONAL $H(z)$ DATA SET AND BAO

### A. The Observational $H(z)$ Data

The Hubble parameter  $H(z)$  depends on the differential age of the Universe in this form

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}, \quad (10)$$

which provides a direct measurement for  $H(z)$  through a determination of  $dz/dt$ . By using the differential ages of passively evolving galaxies determined from the Gemini Deep Survey Survey (GDDS) [29] and archival data [30, 31, 32, 33], Simon et al. determined a set of observational  $H(z)$  data in the range  $0 \lesssim z \lesssim 1.8$  and used them to constrain the dark energy potential and its redshift dependence [34]. Using this data set, one can constrain parameters of various cosmological models. Yi & Zhang first used them to analyze the holographic dark energy models in which the parameter  $c$  plays a significant role [24]. The cases with  $c = 0.6, 1.0, 1.4$  and setting  $c$  free are discussed in detail and the results are consistent with others. Samushia & Ratra used this data set to constrain the  $\Lambda$ CDM, XCDM and  $\phi$ CDM models [35] and Wei & Zhang analyzed a series of other cosmological models with interaction between dark matter and dark energy [36]. But as pointed out by Wei & Zhang, the data point near  $z \sim 1.5$  derives from the main trend seriously and dip down sharply [36]. We will omit this point in later discussion.

## B. The BAO Data

The acoustic peaks in the CMB anisotropy power spectrum has been found efficient to constrain cosmological parameters [1]. Using a large spectroscopic sample of 46,748 luminous red galaxies covering 3816 square degrees out to  $z = 0.47$  from the SDSS, Eisenstein et al. successfully found the peaks, described by A-parameter that is independent of the dark energy models[26],

$$A = \frac{\sqrt{\Omega_m}}{z_1} \left[ \frac{z_1}{E(z_1)} \frac{1}{|\Omega_k|} \text{sinn}^2(\sqrt{|\Omega_k|} F(z_1)) \right]^{1/3}, \quad (11)$$

where  $E(z) = H(z)/H_0$ ,  $z_1 = 0.35$  is the redshift at which the acoustic scale has been measured, the function  $\text{sinn}(x)$  is defined as

$$\text{sinn}(x) \equiv \begin{cases} \sinh(x) & \text{if } \Omega_k > 0; \\ x & \text{if } \Omega_k = 0; \\ \sin(x) & \text{if } \Omega_k < 0, \end{cases} \quad (12)$$

and the function  $F(z)$  is defined as

$$F(z) \equiv \int_0^z \frac{dz}{E(z)}. \quad (13)$$

Eisenstein et al. suggested the measured value of the A-parameter as  $A = 0.469 \pm 0.017$  [26, 27]. For more information on BAO, see [26]. The BAO data has been widely used as a test for cosmological parameters. Wu & Yu combined BAO with some recent observational data to determine parameters of a dark energy model with the equation of state  $\omega = \omega_0/[1 + b \ln(1 + z)]^2$  [37]. Su et al. combined BAO with GRBs to analyze the  $\Lambda$ CDM cosmological model [38]. It has been declaimed that BAO is quite robust to constrain cosmological parameters.

More seriously, the BAO data is used only through the fitting formula given by Eisenstein et al[26]. However, the A-parameter has been tested only within the limited framework of standard  $\Lambda$ CDM (i.e., dark energy models), so it is just independent of the dark energy models not completely model-independent. Moreover, the growth of perturbations in the DGP model is also not the same as that in  $\Lambda$ CDM [39], and therefore the baryon acoustic peak in the DGP model cannot just be located on the same scale as that in  $\Lambda$ CDM. Even so, the discrepancy between the DGP model and dark energy models do not affect the constraints on the DGP model using BAO data. Guo et al made the constraints the DGP model using recent supernova observations and BAO[20], and Pires et al also used BAO to make a joint statistics for the DGP braneworld cosmology with the lookback time data set[19].

## IV. CONSTRAINTS ON THE DGP UNIVERSE

First we study the case with a curvature term and assume a prior of  $h = 0.73 \pm 0.03$  from the combinational WMAP three-year estimate [1]. In order to estimate the best-fit values of  $\{\Omega_m, \Omega_{r_c}\}$ , we use the standard  $\chi^2$  minimization method. If we use only the observational  $H(z)$  data set, we get the fitting results  $\Omega_m = 0.71 \pm 0.16$  and  $\Omega_{r_c} = 0.30 \pm 0.40$ . The best-fit values correspond to a closed and accelerating Universe with  $\Omega_k = -1.41$  and  $q_0 = -0.77$ . The current value of the effective equation of state is  $\omega_{\text{eff}} = -0.78$ . This constraint seems very weak due to the large values of the  $1\sigma$  errors and requires combinational analysis with other cosmological probes. If we combine the observational  $H(z)$  data with BAO, we get  $\Omega_m = 0.30 \pm 0.02$  and  $\Omega_{r_c} = 0.14 \pm 0.03$ . The best-fit values suggest a closed and accelerating Universe too, with  $\Omega_k = -0.08$  and  $q_0 = -0.37$ . The current value of the effective equation of state is  $\omega_{\text{eff}} = -0.78$ .

TABLE I: Fitting results for the corresponding parameters for a non-flat DGP Universe with a prior  $h = 0.73 \pm 0.03$ 

Test	$\Omega_m$	$\Omega_{r_c}$	$r_c^a$	$q_0$	$z_{tr}$	$\omega_{eff0}$
OHD	$0.71 \pm 0.16$	$0.30 \pm 0.40$	0.91	-0.77	0.50	-0.78
OHD+BAO	$0.30 \pm 0.02$	$0.14 \pm 0.03$	1.34	-0.37	0.55	-0.78

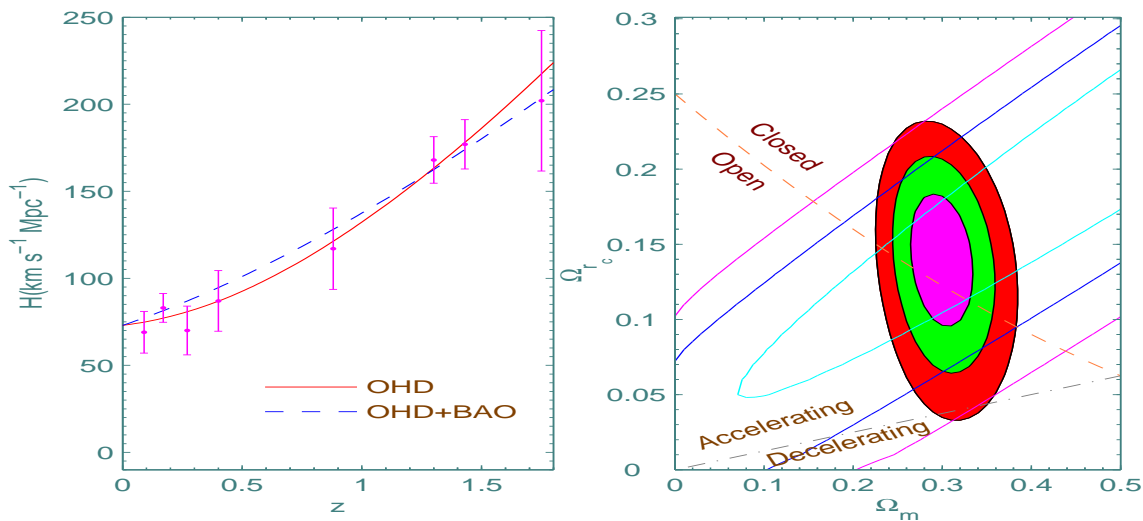
<sup>a</sup>in units of  $H_0^{-1}$ 

FIG. 1: Constraints from the observational  $H(z)$  data (OHD) and the BAO data for a non-flat DGP Universe. The left panel:  $H(z)$  as a function of  $z$  with the best-fit values of  $\Omega_m$  and  $\Omega_{r_c}$ , and the observational data with  $1\sigma$  error bars are also plotted. The right panel: Confidence regions in the  $\Omega_m - \Omega_{r_c}$  plane for the joint analysis of OHD+BAO (the shaded regions from inner to outer stand for confidence levels of 68.3%, 95.4% and 99.7% respectively), as well as analysis of only OHD (the solid lines from inner to outer stand for confidence regions of 68.3%, 95.4% and 99.7% respectively).

The two cases provide nearly the same evolutionary values of  $\omega_{eff}$ . All the best-fit results are listed in Table I, as well as  $q_0$ ,  $z_{tr}$  and  $\omega_{eff0}$ . In the left panel of Fig.1, we plot  $H(z)$  as a function of  $z$  using the best-fit results for the two cases. And we present the confidence regions in the  $\Omega_m - \Omega_{r_c}$  plane in the right panel of Fig.1, for cases with and without the BAO data. An accelerating Universe is suggested at  $3\sigma$  confidence level. Larger regions correspond to a closed Universe even though an open Universe is possible.

We will look into the flat DGP Universe and we set  $h$  free instead of taking a prior. If we use only the observational  $H(z)$  data, we get the fitting results  $h = 0.67 \pm 0.07$  and  $\Omega_{r_c} = 0.10 \pm 0.04$ . The best-fit values correspond to an accelerating Universe with  $q_0 = -0.19$ . And the current value of the effective equation of state is  $\omega_{eff0} = -0.73$ . If we combine BAO to make a combinational analysis, we get  $h = 0.70 \pm 0.03$  and  $\Omega_{r_c} = 0.12 \pm 0.09$ . The best-fit values suggest an accelerating Universe with  $q_0 = -0.29$ . And the current value of the effective equation of state is  $\omega_{eff0} = -0.76$ . The values of  $\omega_{eff0}$  for the two cases are close to each other. All the best-fit results are listed in Table II, as well as  $\Omega_m$ ,  $q_0$ ,  $z_{tr}$  and  $\omega_{eff0}$ . In the left panel of Fig.2, we plot  $H(z)$  as a function of  $z$  using the best-fit results for the two cases. The confidence regions in the  $\Omega_{r_c} - h$  plane are presented in the right panel of the same figure, for cases with and without the BAO data. An accelerating Universe is suggested at  $3\sigma$  confidence level for combining OHD and BAO.

## V. DISCUSSIONS AND CONCLUSIONS

Various cosmological observations have been used to explain the acceleration of the DGP Universe. In this work, we constrain the cosmological parameters from the observational  $H(z)$  data set and the BAO data. The current values of the deceleration parameter  $q_0$ , the transition redshift  $z_{tr}$  and the current value of the effective equation of state  $\omega_{eff0}$  have been derived too. Indeed, the acceleration seems clear for both the cases with and without a curvature

TABLE II: Fitting results for the corresponding parameters for a flat DGP Universe

Test	$h$	$\Omega_{r_c}$	$\Omega_m$	$r_c^a$	$q_0$	$z_{tr}$	$\omega_{eff0}$
OHD	$0.67 \pm 0.07$	$0.10 \pm 0.04$	0.37	1.58	-0.19	0.29	-0.73
OHD+BAO	$0.70 \pm 0.03$	$0.12 \pm 0.09$	0.31	1.44	-0.29	0.46	-0.76

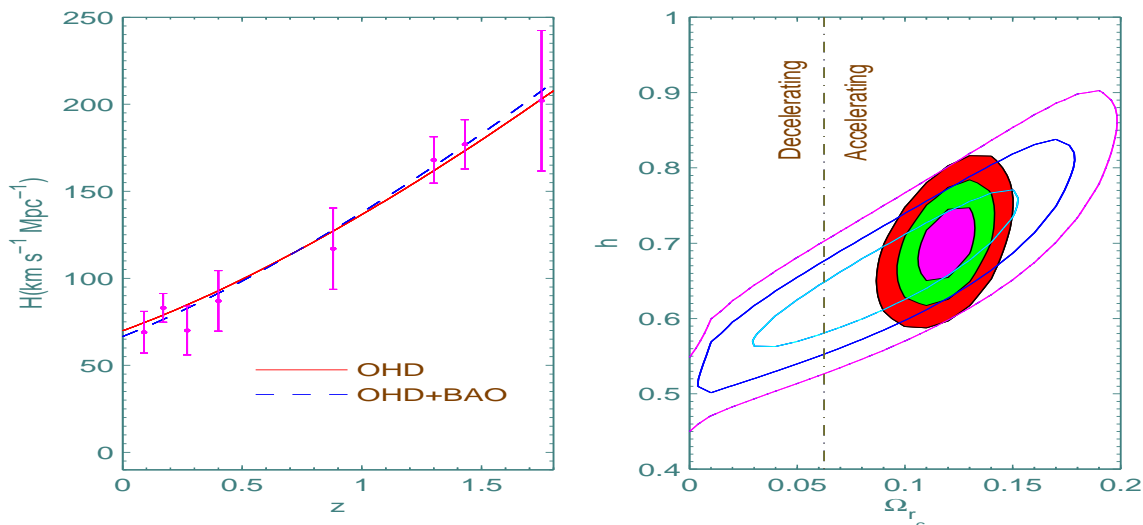
<sup>a</sup>in units of  $H_0^{-1}$ 

FIG. 2: Constraints from the observational  $H(z)$  data (OHD) and the BAO data for a flat Universe. The left panel:  $H(z)$  as a function of  $z$  with the best-fit values of  $\Omega_m$  and  $\Omega_{r_c}$ , and the observational data with  $1\sigma$  error bars are also plotted. The right panel: Confidence regions in the  $\Omega_{r_c} - h$  plane for the joint analysis of OHD+BAO (the shaded regions from inner to outer stand for confidence levels of 68.3%, 95.4% and 99.7% respectively), as well as analysis of only OHD (the solid lines from inner to outer stand for confidence regions of 68.3%, 95.4% and 99.7% respectively).

term. For the former case, the best-fit results correspond to a closed Universe although an open Universe is possible at larger confidence levels. This is consistent with many other constraint conclusions [19, 40, 41, 42]. And the current values of  $\omega_{eff0}$  are close to each other for either combining BAO or not. For the flat Universe, values of  $h$  are a little smaller than the result from the combinational WMAP three-year estimate [1]. But they are consistent with the result  $h = 0.68 \pm 0.04$  suggested from a media statistics analysis of the current value of the Hubble parameter [43, 44]. To make a comparison, we use the same data to do a constraint on the standard  $\Lambda$ CDM cosmological model. In Table III, we list the values of  $\Delta\chi^2(\text{DGP-}\Lambda\text{CDM})$ , which is the excess  $\chi^2$  value between the best-fit DGP Universe and that of  $\Lambda$ CDM. The DGP Universe and the  $\Lambda$ CDM Universe have the same degrees of freedom no matter whether a curvature term is included. For the DGP Universe with a curvature term (non-flat), this value is a little larger than *zero*, which means that the fit for the DGP Universe is a little poorer than  $\Lambda$ CDM. For the flat DGP Universe,  $\Lambda$ CDM is fit better if only the observational  $H(z)$  data are used, while this is greatly reversed if the two data sets are considered together. In one word, the observational  $H(z)$  data set can be seen as an acceptable cosmological probe. And the DGP Universe does not contradict with most observational results. But as the amount of the observational  $H(z)$  data is still so few, there exist many deficiencies waiting for improving. Combinational analysis with other observations such as the BAO data is an efficient way which can provide stronger constraint on the cosmological parameters.

TABLE III: The values of  $\Delta\chi^2(\text{DGP-}\Lambda\text{CDM})$ 

Test	non-flat	flat
OHD	0.006	0.123
OHD+BAO	0.130	-1.744

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